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ABSTRACT

The objectives of this work are 1) to develop a robust stability theory for hybrid dynamical systems, both in continuous time and discrete time, including a complete theory on the existence of smooth Lyapunov functions (which are the major workhorse for the analysis of non-hybrid dynamical systems) for hybrid systems; 2) to exploit this theory in developing tools for the design of robust feedback control algorithms for hybrid dynamical systems; and 3) to apply the developed control concepts to problems relevant to the Army's mission, ie.g., autonomous vehicles and network centric control.

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(a) Papers published in peer-reviewed journals (N/A for none)

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Chaohong Cai, R. Goebel and A.R. Teel, "Smooth Lyapunov functions for hybrid systems, Part I: Existence is equivalent to robustness", IEEE Transactions on Automatic Control, to appear, 2007.

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Number of Papers published in peer-reviewed journals: 8.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

A.R. Teel, "Robust hybrid control systems: an overview of some recent results", In Advances in Control Theory and Applications, Lecture Notes in Control and Information Sciences , Vol. 353, Springer, to appear, March 2007, Springer.

R. Goebel, T.. Hu, A.R. Teel, "Dual matrix inequalities in stability and performance analysis of linear differential/difference inclusions", In Current Trends in Nonlinear Systems and Control, In honor of Petar Kokotovic and Turi Nicosia, Menini, Zaccarian, Abdallah, eds. pp. 103-122, Birkhauser, 2006.

Number of Papers published in non peer-reviewed journals: 2.00

(c) Presentations

J. Hespanha and A.R. Teel, "Modeling, analysis and design of hybrid control systems", 17th International Symposium on Mathematical Theory of Networks and Systems Kyoto International Conference Hall, Kyoto, Japan, July 24-28, 2006, (1 of 3 mini-courses).

C. Cai, R. Goebel, R. Sanfelice, and A.R. Teel, "Robust hybrid systems: theory and applications", 45th IEEE Conference on Decision and Control, San Diego, CA, December 2006, pre-conference workshop.

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D. Carnevale, A. R. Teel, D. Nesic, "Further results on stability of networked control systems: a Lyapunov approach", Proceedings of the 2007 American Control Conference, to appear.

L. Zaccarian, D. Nesic, A. R. Teel, "Set-point stabilization of SISO linear systems using First Order Reset Elements", Proceedings of the 2007 American Control Conference, to appear.

C. Cai, A. R. Teel, R. Goebel, "Results on existence of smooth Lyapunov functions for hybrid systems with nonopen basin of attraction", Proceedings of the 2007 American Control Conference, to appear.

R. G. Sanfelice, A. R. Teel, "A `throw-catch' hybrid control strategy for robust stability of nonlinear systems", Proceedings of the 2007 American Control Conference, to appear.

P. Naghshtabrizi, J. P. Hespanha, A. R. Teel, "Stability of Infinite-Dimensional Impulsive Systems with Application to Network Control Systems", Proceedings of the 2007 American Control Conference, to appear.

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"Converse Lyapunov theorems and robust asymptotic stability for hybrid systems", Chaohong Cai; Teel, A.R.; Goebel, R., Proceedings of the 2005 American Control Conference, June 8-10, 2005, Page(s): 12- 17.

"Results on convergence in hybrid systems via detectability and an invariance principle", Sanfelice, R.G.; Goebel, R.; Teel, A.R., Proceedings of the 2005 American Control Conference, June 8-10, 2005,
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"Results on solution sets to hybrid systems with applications to stability theory", Goebel, R.; Teel, A.R., Proceedings of the 2005 American Control Conference, June 8-10, 2005, Page(s): 557- 562.

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"Hybrid systems: generalized solutions and robust stability", R. Goebel, J. Hespanha, A.R. Teel, C. Cai and R. Sanfelice, Proceedings of the 2004 IFAC Symposium on Nonlinear Control Systems Design, Stuttgart, Germany, vol. 1, pp. 1-12.

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(d) Manuscripts

C. Cai, R. Goebel, A.R. Teel, "Smooth Lyapunov functions for hybrid systems, Part II: (pre-)asymptotically stable compact sets", submitted to IEEE Transactions on Automatic Control, 2006.

D. Carnevale and A. R. Teel and D. Nesic, "A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems", submitted to IEEE Transactions on Automatic Control, 2006.

R.G. Sanfelice, R. Goebel, and A.R. Teel, "Generalized solutions to hybrid dynamical systems", submitted to ESAIM:COCV (Control, Optimisation, and Calculus of Variations), 2006.

R.G. Sanfelice, R. Goebel, and A.R. Teel, "Invariance principles for hybrid systems with connections to detectability and asymptotic stability", submitted to IEEE Transactions on Automatic Control, 2006.

Christophe Prieur, Rafal Goebel, and Andrew R. Teel, "Hybrid feedback control and robust stabilization of nonlinear systems", Submitted to IEEE Transactions on Automatic Control, 2006.

Number of Manuscripts: 5.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED	
Chaohong Cai	1.00	No
FTE Equivalent:	**1.00**	
Total Number:	**1**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Andrew R. Teel	0.08	No
Joao P. Hespanha	0.08	No
FTE Equivalent:	**0.16**	
Total Number:	**2**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Personnel receiving masters degrees

NAME	
Chaohong Cai	No
Total Number:	**1**

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME PERCENT_SUPPORTED

FTE Equivalent:

Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Summary statement for
A robust stability and control theory for hybrid dynamical systems
Proposal Number 44527-CI
Professors Andrew R. Teel & Joao P. Hespanha, University of California, Santa Barbara

Statement of problem studied

The work carried out under this grant addressed, first, the task of developing a theory for hybrid dynamical systems that lends itself naturally to classical asymptotic stability concepts and to simple robustness assessments. Second, it looked at exploiting this theory to develop new, robust feedback control algorithms, using hybrid feedback and/or controlling hybrid systems. (Hybrid dynamical systems are those with state variables that can flow and also jump.) Third, it considered applying the developed control concepts to problems relevant to the Army's mission, including autonomous vehicles and network centric control.

Hybrid dynamical systems have appeared in the systems and control literature over at least the last forty years, with intense focus over the last ten years. Relevant literature includes work on impulsive systems and also hybrid automata. While considerable attention has been given to describing solutions of hybrid systems and developing stability analysis tools, including Lyapunov-like conditions for asymptotic stability, to the best of the investigators' knowledge, robustness in hybrid dynamical systems had not been addressed in a systematic manner.

For continuous-time or discrete-time systems, nominal robustness properties have been established and are, by now, very well understood. For a differential equation with a continuous right-hand side, when a compact set is asymptotically stable it is necessarily robustly asymptotically stable. This means that small perturbations to the right-hand side of the differential equation (which might arise in a control system due to measurement noise or other exogenous signals, or due to mismatch between a plant model and the plant's real behavior), do not change the qualitative behavior of the system. For a difference equation with a continuous right-hand side, the same result applies. In both cases, this robustness is crucial for the success of a control system, since exogenous disturbances and plant/model mismatch are ubiquitous.

It may also be noted that a parallel robustness result exists for *discontinuous* differential and difference equations. In particular, such systems admit a generalized notion of solution, due to Krasovskii and other researchers, which finds its utility in the following fact: if a compact set is asymptotically stable when generalized solutions are considered then robust asymptotic stability accrues.

Despite the fact that hybrid systems can be viewed as a combination of differential equations and difference equations, where robustness results hold, no general robustness results for asymptotically stable hybrid systems had appeared in the literature before the work undertaken in this grant.

Lyapunov functions have appeared frequently in the development of stability analysis tools for hybrid dynamical systems. Many different sufficient conditions for asymptotic stability have been given in the literature. Often these conditions weaken the standard decrease conditions that a Lyapunov function should satisfied. Instead, the Lyapunov function may be allowed to increase at some points in time, as long as the increase is balanced by a decrease at other times. Until the work done under this grant, it was an open question as to whether an asymptotically stable hybrid system always admitted a Lyapunov function in the classical sense: a continuously differentiable function that always decreases during flows and always decreases during jumps. In continuous-time and discrete-time systems, the literature has pointed out a direct link between the existence of such functions and robust asymptotic stability. Thus, a theory on robustness of asymptotic stability in hybrid systems was expected to go hand-in-hand with the development of "converse Lyapunov theorems" for hybrid systems. An affirmative answer to the question about the existence of Lyapunov functions for hybrid systems would be expected to spawn a more aggressive search for such functions for particular systems, and to give solid footing to the development of Lyapunov-based control ideas for the synthesis of hybrid feedback for classical or hybrid dynamical systems.

Even with an answer to the "converse" Lyapunov question, it is acknowledged that it is desirable to develop sufficient conditions for asymptotic stability that weaken the requirements on a Lyapunov function while still requiring minimal knowledge about the solutions to the hybrid system. In this direction, simple "LaSalle"-type asymptotic stability results have appeared in the hybrid systems literature. In these results, the Lyapunov function is not required to be strictly decreasing. Instead, it is supposed to be non-increasing and trajectories are then known to converge to the largest invariant set where the function remains constant. Some of the main limitations of the results that have appeared in the literature in this direction are that they apply only to hybrid systems that admit unique solutions which are continuous, in an appropriate sense. These assumptions are very restrictive for hybrid systems and results are desired where such assumptions are not required.

In addition to tools that can be used to establish asymptotic stability in hybrid systems, the work in this grant also considered results that can be used to establish certain input-output stability properties, such as input-to-state stability.

On the feedback control synthesis side, the work in this grant looked at making systematic control design procedures that rely on logic variables, including hysteresis-switching logic. In this context, it also considered Lyapunov-based hybrid feedback design. Moreover, it considered solving certain nonlinear control problems with hybrid feedback where topological constraints make it impossible to solve the problem using non-hybrid feedback.

Summary of the most important results

In the first major contribution from the work on this grant, we developed a framework for understanding the solutions to hybrid dynamical systems which lends itself naturally to robustness results. Building upon the work of other researchers, we codified the notion of a hybrid time domain and showed, under very minimal assumptions (see next paragraph), that each subsequence of solutions to a hybrid system with converging initial condition yields a subsequence converging to a solution of the hybrid system. The convergence is in a graphical sense where each solution together with its hybrid time domain forms a set of points that converges (in the sense of set convergence) to the limiting solution.

In order to get this "sequential compactness" of solutions to hybrid systems, which is crucial for establishing robustness properties, minimal assumptions on the data of the hybrid system are needed. First, in the case where the flow map and jump map are single-valued functions, they should be continuous, as in the case of purely continuous-time or discrete-time systems. Second, the set of points where flowing is possible should be closed and the set of points where jumping is possible should be closed. Under these assumptions, we showed that sequential compactness holds. In turn, this result was used to show several new, fundamental facts related to stability theory for hybrid dynamical systems. First, the basin of attraction for an asymptotically stable compact set in a hybrid system is open relative to the union of the flow set and the jump set. Second, the convergence toward the compact set is uniform over compact subsets of the basin of attraction. Third, asymptotic stability is automatically robust to small perturbations. This "total stability" result generates hybrid versions of many classical robustness results, like robustness to slowly-varying and/or "weakly jumping" parameters, as well as new robustness results like robustness to sufficiently large average dwell-time constraints on jump times, etc.

In the case where the data of the hybrid system does not satisfy these basic conditions, we introduced the notion of generalized solutions to a hybrid system, paralleling the notion introduced by Krasovskii for discontinuous differential equations. We showed that every generalized solution could be reproduced, with arbitrary precision, on compact hybrid time domains by using arbitrarily small perturbations to the data of the hybrid system. In this sense, we established that generalized solutions do not introduce any "spurious" solutions. All generalized solutions have meaning in terms of solutions that can appear under arbitrarily small perturbations. Moreover, we established that if a set is asymptotically stable when generalized solutions are considered then the set is robustly asymptotically stable. The main message from these results is that one should consider modeling hybrid systems using closed flow and jump sets from the beginning. This is because it is only when one does this that one is guaranteed that the observed behavior of the system is robust to perturbations.

We went on to establish that every asymptotically stable hybrid system admits a smooth Lyapunov function, one that decreases in every possible flow direction at every point in the flow set and that decreases due to every possible jump from every point in the jump

set. Then these results were extended to hybrid systems that are input-to-state stable, which is a stability concept that qualifies the effect of disturbances on the state of the system and generalizes asymptotic stability.

On the side of sufficient conditions for asymptotic stability, we developed the most general version of LaSalle's invariance principle that is available for hybrid dynamical systems. In our result, uniqueness of solutions is not needed (this relaxation turns out to be critical in hybrid control systems that produce decision boundaries separating one outcome from another outcome, for example in an automated traffic system where a controller makes a decision about whether to proceed through a yellow light based on current speed and traffic conditions), and *non-smooth* Lyapunov-like functions are admissible. The development of this tool makes it easier to establish asymptotic stability in hybrid control systems.

With a clearer understanding of the behavior of hybrid systems and of the results pertaining to stability, we were equipped to make advances in the development of hybrid control systems. In particular, we were able to establish the advantages of using hybrid control. For example, we showed that hysteresis switching could be used systematically to robustly solve decision-making control problems where topological constraints prohibit a robust solution using non-hybrid feedback. For example, the problem of globally stabilizing the orientation of a vehicle with constant velocity to a pre-specified orientation (which amounts to global stabilization of a point on a circle) requires hybrid feedback to produce a solution that is robust to small measurement noise. In addition, we showed that hysteresis switching could also be used systematically to achieve robust asymptotic stability for every nonlinear control system that is asymptotically controllable. In turn, this result led us to develop the novel notion of a patchy control Lyapunov function (we showed that every asymptotically controllable nonlinear system admits a smooth, patchy control Lyapunov function) and to give procedures for specifying hybrid feedback stabilizers from patchy control Lyapunov functions.

We also provided new insight into the synthesis of reset control systems. The first example of such a system involved the "Clegg integrator", developed in 1958 having the property that the output was the integral of the input but the output was reset to zero whenever the input to the integrator passed through zero. We used our understanding of hybrid systems to point out that certain existing models of reset control systems that had appeared in the literature were not well-posed and that working with a well-posed model, which we proposed, led to far less conservative characterizations of asymptotic stability in reset control systems.

We studied networked control systems from a hybrid systems point of view and have reported some of the least conservative quantifications of network-induced delay that is admissible for asymptotic stability in networked control systems. Several of these results rely on very simple Lyapunov-based proofs, inspired by our results on converse Lyapunov theorems.

We have also initiated an investigation of using hybrid (logic-based) feedback to orchestrate the knowledge of open-loop controls, which steer a system from one configuration to another configuration, with local feedback control to achieve global stabilization for challenging control problems. A prototypical example is global stabilization of the upright position of a multi-link pendulum (which, due to topological constraints, cannot be solved robustly using non-hybrid feedback). In such a system, it is easy to build a feedback stabilizer that brings the system to one of several possible rest configurations; from each of these rest configurations it is easy to steer the system to the straight-down position, which is a point that can be easily stabilized locally; from the straight-down position it is relatively easy to throw the pendulum to near the upright position using open-loop control; finally it is possible to locally stabilize the straight-up position. All of these components can be coordinated, using hybrid feedback, to achieve global stabilization. We call this idea "throw-and-catch" control. We are just beginning to explore its significance for other control problems, such as those where vehicles pass through regions where position measurements are not available and open-loop control signals are used to steer from one region with sensor coverage to a different region with sensor coverage.

As another control application, we have developed hybrid control strategies for an autonomous vehicle, perhaps constrained with constant velocity, to locate a source using measurements only of the field intensity of the source. For this problem, we have shown how the use of logic-based feedback enables using simple, off-the-shelf, optimization algorithms within the context of dynamical systems to achieve source localization.

Overall, we feel that our improved understanding of stability theory and robustness in hybrid dynamical systems has positioned us to make many significant strides in the development of hybrid control algorithms, and that the control developments reported above are just scratching the surface of what is possible in this arena.

We believe the contributions in the area of solution description and a robust stability theory are fundamental and will find widespread application as the area of hybrid systems grows.